No.	B.D.	R.A. 1900	Decl.	P.	D.	Mags.		Nights.	Date.
		h m	° ′	°	″				1911
1118	+51° 3468	22 43·3	+51 39	131·6	6·67	9·5	11·3	2	·672
1119	+51° 3509	52·7	51 36	115·8	4·51	9·4	9·8	2	·703
1120	+50° 3893	53·8	51 1	23·2	7·97	9·3	9·4	3	·774
1121	+51° 3524	22 56·9	52 7	331·9	1·18	9·3	9·5	3	·655
1122	+50° 3940	23 1·7	50 38	83·5	2·50	9·2	9·3	2	·891
1123	+51° 3565	10·4	52 2	309·6	2·53	9·4	10·0	3	·691 AB
				125·9	17·37	C=12·3		3	·691 AC
1124	+50° 4164	45·3	50 41	246·7	2·32	9·2	9·4	3	·931
1125	+50° 4184	23 47·4	+51 6	333·8	5·05	9·5	9·6	3	·838

Notes.

1077. The star B.D. + 50°1344 was found to be 13 mag. on Jan. 31, and is not identical with the pair here measured.

1121. Professor Fox kindly measured this object and obtained :—

1911·709 P 329°·4 D 1″·34 3 nts.

The Absorption of Light in Space. By Francis G. Brown.

(*Communicated by S. A. Saunder.*)

For some time past I have devoted some attention to the problem of the absorption of light in space. The researches of M. Tikhoff, Professor Turner, Professor Kapteyn, and others have indicated that some absorption exists, but the results have not been regarded as conclusive. It occurred to me that it might be possible to use the nebulæ in a determination of the extent of this absorption. Unlike the fixed stars, the nebulæ have a measurable diameter, and it is therefore possible to ascertain the relative distances of various groups of nebulæ, since it follows that those objects with a small apparent diameter must be, *on the average*, more distant than the larger objects, in spite of the fact that their real diameters vary greatly. If there were no absorption of light in its passage through space, the intrinsic, or surface, brightness of a nebula would be the same whatever its distance from the Earth; on the other hand, if the light were partly absorbed, the average brightness of the small nebulæ would be less than that of the larger ones.

The method of classification adopted in my investigations was as follows: using the descriptions in the New General Catalogue of Nebulæ, which are sufficiently exact when a large number of objects are dealt with, I separated the various classes marked

therein as "very large," "large," etc., and for the described brightness, substituted a number according to the following scale:—

*e*B = 9		*p*B = 5
*v*B = 8		*p*F = 4
B = 7		F = 3
*c*B = 6		*v*F = 2
	*e*F = 1	

Classifying all the nebulæ down to those marked "very small" in the New General Catalogue, the following results are arrived at:—

TABLE I.

Average Brightness of the Nebulæ.

Class.	Approximate Diameter.	No. of Objects.	Total of the Numerical Substitutions for Brightness.	Average for Class.
*v*L	8′–10′	156	800	5·13
L	4′	325	1359	4·18
*c*L	3′	164	671	4·09
*p*L	1′–2′	832	2917	3·50
*p*S	30″–1′	998	2966	2·98
S	20″–30″	1975	5379	2·72
*v*S	10″–12″	1325	2609	1·97

One considerable disadvantage in the employment of this method arises from the fact that the existing catalogue brightness is not always reliable, owing to a tendency to over-estimate the brightness of a large diffused nebulæ, and to under-estimate that of a small condensed object. The differences between successive classes in the above table are, however, so striking, that it is evident that the greater part of the decrease is real. With more accurate data as to the brightness of the nebulæ, it should be possible to obtain a reliable determination of the amount of absorption for each class.

In the above classification I kept each hour of Right Ascension separate, and the results studied in this way are interesting. Table II. shows the average brightness of the various classes, together with certain combinations of classes over each of the twenty-four hours of right ascension. It will be seen that there is considerable variation in the brightness in successive hours, and this is noticeable not only in the case of the large nebulæ, which, being comparatively few in number, naturally show certain fluctuations, but also in the case of the smaller classes. If we calculate the harmonics for the groups *v*L – *p*L and *p*S – S, which are, of course, quite independent, we get the following, ϕ denoting the R.A. counted from the middle of group I. as zero: *i.e.* from R.A. $0^h\ 30^m$.

For vL − pL : $+0{\cdot}38 \operatorname{Sin}\phi - 0{\cdot}14 \operatorname{Cos}\phi + 0{\cdot}29 \operatorname{Sin} 2\phi + 0{\cdot}04 \operatorname{Cos} 2\phi$
$+ 0{\cdot}11 \operatorname{Sin} 3\phi - 0{\cdot}36 \operatorname{Cos} 3\phi$

For pS − S : $+0{\cdot}25 \operatorname{Sin}\phi - 0{\cdot}26 \operatorname{Cos}\phi - 0{\cdot}13 \operatorname{Sin} 2\phi + 0{\cdot}01 \operatorname{Cos} 2\phi$
$- 0{\cdot}13 \operatorname{Sin} 3\phi - 0{\cdot}30 \operatorname{Cos} 3\phi$

TABLE II.

Mean Brightness of the various classes of Nebulæ in Right Ascension.

Hour.	vL.	L.	cL.	pL.	pS.	S.	vS.	vL − pL.	pS − S.	vL − S.	vL − vS.
I.	5·57	4·67	2·16	3·08	2·82	2·34	1·80	3·62	2·47	2·78	2·49
II.	5·20	4·08	4·14	3·65	1·92	2·58	2·02	3·87	2·41	2·89	2·63
III.	7·25	4·00	3·83	3·00	2·25	2·51	1·79	3·67	2·42	2·73	2·45
IV.	4·50	5·23	5·11	3·73	3·33	2·55	1·82	4·40	2·77	3·28	2·90
V.	6·57	5·64	5·00	3·44	3·31	2·86	1·73	4·56	3·03	3·52	3·07
VI.	6·33	4·91	3·75	3·53	2·88	3·50	3·30	5·19	3·30	3·69	3·66
VII.	—	4·00	3·25	3·59	3·36	2·81	1·85	3·57	2·98	3·23	2·91
VIII.	5·50	1·75	3·50	4·00	2·40	3·06	1·64	3·61	2·91	3·11	2·68
IX.	5·67	3·37	3·14	3·14	2·48	2·65	1·75	3·37	2·60	2·84	2·60
X.	5·33	5·00	4·20	2·90	3·07	2·62	2·04	3·74	2·78	3·08	2·88
XI.	3·85	4·64	5·00	3·24	2·83	2·79	2·07	3·94	2·80	3·22	3·05
XII.	5·86	4·33	5·22	4·22	3·60	3·00	1·96	4·57	3·22	3·70	3·27
XIII.	5·35	4·24	4·50	3·89	3·55	3·08	2·24	4·26	3·24	3·69	3·48
XIV.	4·90	3·92	4·22	3·83	3·25	2·94	2·14	3·97	3·05	3·39	3·14
XV.	3·25	3·94	3·60	3·80	3·03	2·50	1·96	3·79	2·70	2·96	2·79
XVI.	3·50	2·80	5·25	3·39	2·89	2·67	1·90	3·53	2·76	2·97	2·71
XVII.	2·50	5·50	3·75	3·15	1·92	2·24	1·84	3·35	2·13	2·48	2·23
XVIII.	5·00	3·28	2·00	3·09	2·37	2·35	1·86	3·38	2·35	2·65	2·41
XIX.	4·25	3·50	6·00	3·35	2·41	2·85	2·48	3·62	2·67	2·92	2·80
XX.	8·00	2·00	4·00	3·50	2·14	3·37	1·89	3·33	2·92	3·04	2·89
XXI.	2·33	4·00	4·25	3·24	2·59	2·94	1·85	3·41	2·81	3·04	2·79
XXII.	1·00	4·50	3·00	3·20	4·00	2·46	1·76	3·28	3·24	3·25	2·93
XXIII.	3·67	3·62	4·67	2·58	2·92	2·48	1·98	2·95	2·63	2·72	2·49
XXIV.	—	3·62	3·69	2·80	3·11	2·65	1·89	3·19	2·80	2·89	2·60

The second harmonics are discordant, but there is fair accord between the first and third.

As it appeared improbable that the variation depended directly on the right ascension, I then divided the sky into 88 areas, distributed as follows:—

8	in the zone	between	N.P.D.	0°	and	30°	each	3 hrs.	of R.A. in extent.
12	,,	,,	,,	30°	,,	55°	,,	2 ,,	,,
24	,,	,,	,,	55°	,,	90°	,,	1 hr. ,,	,,
24	,,	,,	,,	90°	,,	125°	,,	1 ,,	,,
12	,,	,,	,,	125°	,,	150°	,,	2 hrs. ,,	,,
8	,,	,,	,,	150°	,,	180°	,,	3 ,,	,,
88									

Table III. shows the mean brightness of the nebulæ in each of the above areas. All the objects in the catalogue were made use of, with the exception of open and globular clusters, and nebulous stars. Where there are less than twenty objects in one area the results for two adjoining regions have been combined.

The distribution of the maximum and minimum regions may be seen in the diagram. There are a number of definite minima shown, and if we travel outward in any direction from these, the average brightness of the nebulæ increases until the zone affected by the next minimum is reached.

It is suggested by this diagram that the phenomenon is due to some physical cause, and not to accidental variation. It remains to be determined whether the variation represents a greater absorption of light in certain parts of the sky, or merely a difference in the actual brightness of the nebulæ in various regions. This may be tested by the star density. If a variable absorption exists, the stars in the regions where the average brightness of the nebulæ is at a minimum will appear more sparsely distributed than elsewhere. I have ascertained the star density near the points of minimum brightness of the nebulæ, and compared it with the density at points with the same galactic latitude where the brightness is approximately at a maximum. The counts made use of are those of Sir William Herschel * for the northern hemisphere, and those of Sir John Herschel † for the southern. The results are shown in Table IV., where the numbers inserted in the columns referring to star density represent the number of stars visible in a field of fifteen minutes diameter with Herschel's twenty-foot reflector.

The evidence of these star counts is in favour of the supposition that there is an increased absorption in the regions indicated on the diagram. Much useful information might be obtained from a comparison of the spectra of stars of similar type in those regions where a maximum or minimum degree of absorption is indicated. Meantime it may help further investigation to reduce Table III. to spherical harmonics, as in the next paper.

* Series I., Gauges 1-683, *Phil. Trans.*, 1785, p. 20. Reprinted by Holden, Publications of the Washburn Observatory, vol. ii., 1883.

Series II., Gauges 684-1088. Reduced by Holden as above.

† *Cape Results*, pp. 375-379.

TABLE III.—*Mean Brightness of the Nebulæ in 74 Areas.*

The upper figures refer to the total brightness, and the lower to the number of nebulæ in the area. Where the number of nebulæ is less than twenty in an area the results for two adjoining regions are combined.

Hour.	N.P.D. 0°–30°.		30°–55°.		55°–90°.		90°–125°.		125°–150°.		150°–180°.	
I.	5/2	Grouped with Hours 22–24.	55/14	2·96	334/147	2·27	335/141	2·37	70/32	2·58	73/23	
II.	0		81/32		488/194	2·47	245/105	2·33	121/42		52/17	3·07
III.	4/2		151/61	2·43	278/124	2·24	405/176	2·30	50/16	3·85	4/2	
IV.	10/3		70/30		42/19		406/164	2·47	196/48		29/8	
V.	16/5		3/1		51/21	2·87	315/122	2·58	154/38	3·68	176/48	3·66
VI.	6/2	3·13	2/1	2·57	38/10		86/35	2·45	41/15		700/192	
VII.	23/7		30/14		39/17	2·09	95/29	3·31	18/7		88/31	
VIII.	29/9		96/35		126/62		44/13		4/1	3·17	25/8	2·90
IX.	76/25		109/40	2·95	201/89	2·26	101/41	2·46	14/5		6/2	
X.	111/29		148/47		349/147	2·38	291/106	2·76	24/6		22/7	
XI.	105/34	3·51	192/61	3·59	309/156	2·50	254/104	2·44	86/23	3·71	13/3	
XII.	135/37		397/103		856/290	2·95	267/105	2·54	44/12		22/7	3·55
XIII.	142/38		341/96	3·49	1689/478	3·53	415/136	3·05	114/39	4·16	0	
XIV.	91/27	3·28	405/118		408/164	2·49	362/122	2·97	104/25		7/1	
XV.	33/16		322/107	2·98	426/163	2·62	155/56	2·77	27/9		11/4	
XVI.	34/12		109/51		227/97	2·34	65/25	2·60	0	3·04	6/2	2·50
XVII.	63/29	2·12	146/72	1·93	131/64	2·04	11/5	4·08	24/7		6/3	
XVIII.	109/56		99/55		118/50	2·36	95/21		22/8		27/11	
XIX.	64/27		54/25	2·37	126/48	2·62	44/13	3·17	27/8	3·23	45/16	
XX.	14/9	2·18	36/13		42/12	2·98	29/10		70/22		23/9	3·00
XXI.	14/6		17/6		104/37		79/37	2·13	88/28	3·61	34/9	
XXII.	9/4		14/4	2·58	99/46	2·15	110/47	2·34	179/46		26/7	
XXIII.	7/1	2·54	51/24		276/131	2·10	172/66	2·61	74/25	3·37	28/7	3·31
XXIV.	3/2		16/4		50/25	2·35	165/79	2·09	88/23		19/8	

Average Brightness of the Nebulæ in 74 Areas.

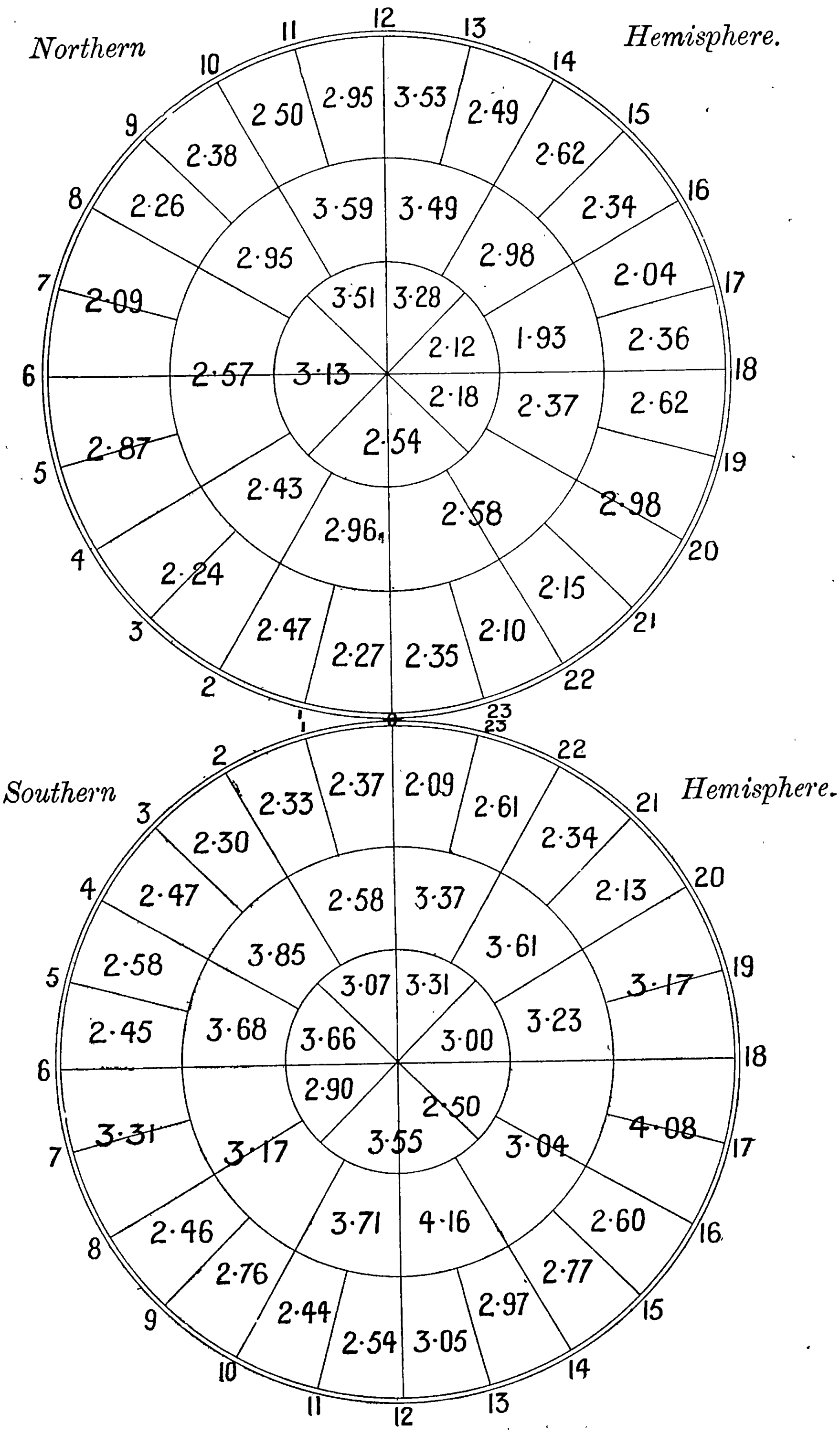

Table IV.

Comparisons of Star Density in Areas of Maximum and Minimum Brightness of the Nebulæ with the same Galactic Latitude.

N.B.—H refers to counts of Sir William Herschel; *h* to those of Sir John Herschel.

Areas of Minimum Brightness.						Areas of Maximum Brightness.					
Position of Area.		Star Density.		Mean Star Density of Area.	Mean Galactic Latitude of Counts.	Position of Area.		Star Density.		Mean Star Density of Area.	Mean Galactic Latitude of Counts.
R.A.	Dec.	Gauge.	No. of Stars.			R.A.	Dec.	Gauge.	No. of Stars.		
h m	°					h m	°				
2·30	+0	H 53	4·9			12 0	+70	H 911	18·2		
		56	6·4					913	10·0		
		58	7·8					920	11·2		
		59	7·2					947	9·3		
		60	4·3								
		69	6·0								
		71	5·6								
		72	6·6								
		75	9·0								
		705	7·5			12·30	−10	H 924	9·6		
		709	7·4					926	10·3		
		77	6·8					931	7·3		
			——	6·6	50°				——	10·8	50°
7·30	+15	H 168	50			6·10	+0	H 137	44		
		179	60					138	60		
		197	48					140	57		
			——	52·0	10°			148	90		
								150	52		
								153	54		
								162	77		
								146	24		
								165	72		
									——	59·0	10°
10·0	+0	H 219	13·8			10–12	+80	H 890	60		
		223	11·5					902	25		
		226	7·9					904	11		
		230	7·2					911	18		
		231	6·5					913	10		
		232	4·9						——	25·0	42°
			——	8·6	37°						

TABLE IV.—*continued.*

Areas of Minimum Brightness.						Areas of Maximum Brightness.					
Position of Area.		Star Density.		Mean Star Density of Area.	Mean Galactic Latitude of Counts.	Position of Area.		Star Density.		Mean Star Density of Area.	Mean Galactic Latitude of Counts.
R.A.	Dec.	Gauge.	No. of Stars.			R.A.	Dec.	Gauge.	No. of Stars.		
h m	°					h m	°				
17·0	+35	H 316	14·2			10 – 12	+80	H 890	60		
		330	15·6					902	25		
			——	14·9	35°			904	11		
								911	18		
								913	10		
									——	25·0	42°
23·15	+ 0	H 669	7·5			11·0	+62	H 911	18·2		
		1071	6·3					932	8·0		
		1072	7·2					949	8·4		
		1081	7·1								
		1083	7·2			12 – 30	– 4	946	7·6		
			——	7·1	56°				——	10·5	60°
21·0	– 15	*h*	14			21·0	– 50	*h*	27		
		,,	11					,,	25		
		,,	12					,,	15		
			——	12·3	30°			,,	17		
									——	21·0	30°
5·30	– 10	*h*	9			5·0	– 50	*h*	11		
		,,	18					,,	9		
		,,	12					,,	17		
		,,	13					,,	17		
			——	13·0	38°			,,	20		
								,,	21		
								,,	9		
								,,	12		
									——	14·5	38°
1·0	– 48	*h*	7			3·0	– 44		2		
		,,	4						10		
		,,	6						10		
		,,	10						6		
		,,	1						12		
		,,	11						13		
			——	6·5	67°				6		
									——	8·4	67°
MEANS :—				16·4	40°					21·8	42°

An example of the use of Spherical Harmonic Analysis.
By H. H. Turner, D.Sc., F.R.S., and F. G. Brown.

1. The use of simple harmonic analysis is only slowly making its way into astronomical computations, although there are many departments in which it is specially suitable, *e.g.* in the observations of variable stars. Even in gravitational astronomy, where the theory and tables have long been expressed in terms of sines and cosines, the observations have seldom been analysed to correspond. Dr. Cowell's analysis of the Greenwich lunar observations is a recent example, but was almost the first.

2. Hence it is scarcely matter for surprise that the rather more elaborate spherical harmonic analysis has made so little headway up to the present. And yet there is special need of it in astronomy for expressing compendiously various distributions over the sphere. Thus we might express the number of stars to the 5th magnitude in this way; and then to the 6th; and to the 7th; and so on: and the run of the coefficients would give us at a glance important information. Presently we may hope to express distributions of proper motions by this method. Again there is the distribution of nebulæ, which can doubtless be shown on a diagram in various ways, but could also be exhibited in a numerical series. Both methods have advantages, and the use of either need not imply disparagement of the other.

3. It seems possible that if the method of spherical harmonic analysis were put into simple and accessible shape, it might be used by some who have hitherto been repelled by the complex appearance of some of the formulæ. The following example is given at some length with this object in view, while at the same time it serves the special purpose of bringing out the main features of the distribution of brightness of nebulæ in different parts of the sphere, given in Table III. of the preceding paper by one of us.

4. On reference to this table it will be seen that the results are collected in zones of N.P.D. (or declination), as will usually be the case in astronomical work. But to avoid seeming to make this necessary, we will use θ for N.P.D., and ϕ for R.A. The first step is to express the results for *each zone separately* in ordinary harmonics; that is, to express each zone in the form

$$a_0 + a_1 \cos\phi + b_1 \sin\phi + a_2 \cos 2\phi + b_2 \sin 2\phi$$
$$+ a_3 \cos 3\phi + a_3 \sin 3\phi + \text{etc.}$$

We must at this point determine how many orders of harmonics we are going to use, and how many figures; for this will characterise all the subsequent work. Experience suggests that at any rate in the first instance we should use *three orders only* (so far as $\cos 3\phi$ and $\sin 3\phi$), and should use *two decimal places* only. In some cases the accordance of the residuals will suggest that we might go further, but little will have been lost by the preliminary work, which will serve as a useful check on the more elaborate investi-